THINKING NOTES

Purpose

Making meaningful notes is a crucial skill for students to develop, but teachers often do not encourage students to make notes or give them ample opportunity to develop as "notemakers." Significant practice time is given to the manipulation of numbers and symbols, as preparation for future problem solving, but little time is afforded to students to make notes about their underlying thinking and reasoning. Thinking Notes places the mechanics of problem solving within a notemaking framework, enabling students to deepen their understanding of important mathematical processes while they develop the skill of making notes.

Overview

For students to produce quality notes, they must elaborate on their work and expose their thinking. Unlike students who passively *take notes* by simply copying the teacher's words and work, students who make notes are actively thinking—they're analyzing the steps of a solution, describing and explaining their work at each step, and capturing questions that arise while they work. As they process steps of the solution, their reasoning is strengthened. The Thinking Notes framework increases student ownership of their learning—at each step in the problem-solving process, students ask and answer the question, "Why did I (take a particular step)?" and thereby focus and deepen their understanding, recalling prior knowledge, patterns, and the goals of distinct steps. They also communicate their reasoning or, if unable to clearly justify why they took a step, they record a question about that step in the process. This step-by-step approach to Thinking Notes works well with the sequential nature of many mathematical processes.

Building Common Core Thinking

Thinking Notes requires students to be deliberate in their problem solving *and* thinking, deepening their understanding while building their confidence for reasoning through problems. Thinking Notes supports the following Standards for Mathematical Practice (MP):

- *(MP 1) **Sense:*** analyzing and explaining transformations of equations
- *(MP 2) **Reason:*** decontextualizing and manipulating symbols and using properties
- *(MP 3) **Argument:*** building logical progressions of statements
- *(MP 6) **Precision:*** clarifying reasoning and formulating explanations
- *(MP 7) **Structure:*** looking closely to discern patterns, stepping back for an overview
- *(MP 8) **Repetition:*** noticing repetitions in computations and similarity in methods

Steps

1. Write out the problem-solving steps for an equation (expression) in the center column of the three-column Thinking Notes form (see Figure 2.13).
2. Model (or review) the Thinking Notes with a sample problem. (*Note:* Until students are very familiar with the Thinking Notes form and process, you should model with a sample problem at least twice. For example, solve an inequality after an equation; perform one of the four operations with fractions, then with mixed numbers.)
3. Once students are comfortable with Thinking Notes, assign a problem for them to solve. (*Note:* Depending on the grade and content, you may want to have students work in small teams to solve a problem initially.)
4. Have students consider each step in the solution and complete the left and right columns of the Thinking Notes form. (*Note:* Encourage students to know and use the names of well-defined properties when writing.)

Examples

Figure 2.13 Student's Completed Thinking Notes Form for *Solving an Inequality*

What Did I Do?	Finding the Solution(s)	Explain Reason(s) Why (or Ask a Question)
Copied original problem	$32 + 3(2x - 12) > 5 - (4 + 9x)$	Starting point of work
Distributed the 3 times (2x - 12) and distributed the minus sign (-1) over (4 + 9x)	$32 + 6x - 36 > 5 - 4 - 9x$	Began simplifying both sides of equation by dealing with "( )" order of operations
Added like terms on both sides	$-4 + 6x > 1 - 9x$	Adding like terms simplifies each side
Add 9x to each side of the problem	$-4 + 15x > 1$	Addition Property of Equality results in having a single x term in equation
Add 4 to each side of the problem	$15x > 5$	Addition Property of Equality isolates the x term on one side
Divide each side by 15	$x > \frac{1}{3}$	Multiplication Property of Inequality helps solve for x

Visit the companion website for a blank "Thinking Notes Organizer."

The following examples show only the solution steps needed for Thinking Notes.

Dividing Mixed Numbers

What Did I Do?	Finding the Solution(s)	Explain Reason(s) Why (or Ask a Question)
	$7\frac{1}{2} \div 2\frac{1}{4}$	
	$\frac{15}{2} \div \frac{9}{4}$	
	$\frac{15}{2} \times \frac{4}{9}$	
	$\frac{5\cancel{15}}{2} \times \frac{4}{\cancel{9}3}$	
	$\frac{5}{1\cancel{2}} \times \frac{\cancel{4}2}{3}$	
	$\frac{10}{3}$	
	$3\frac{1}{3}$	

Trigonometric Identities

What Did I Do?	Finding the Solution(s)	Explain Reason(s) Why (or Ask a Question)
	$\cos^4\Theta - \sin^4\Theta = 1 - 2\sin^2\Theta$	
	$(\cos^2\Theta + \sin^2\Theta)(\cos^2\Theta - \sin^2\Theta) = 1 - 2\sin^2\Theta$	
	$(1)(\cos^2\Theta - \sin^2\Theta) = 1 - 2\sin^2\Theta$	
	$\cos^2\Theta - \sin^2\Theta = 1 - 2\sin^2\Theta$	
	$(1 - \sin^2\Theta) - \sin^2\Theta = 1 - 2\sin^2\Theta$	
	$1 - 2\sin^2\Theta = 1 - 2\sin^2\Theta$	